Rolling the Dice on Big Data

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The Economist, 27 February 2010

Science, 11 February 2011

McKinsey Global Institute, May 2011

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May 2011

Big data: The next frontier for innovation, competition, and productivity


Big data—a growing torrent
$600 to buy a disk drive that can store all of the world's music
5 billion mobile phones in use in 2010
30 billion pieces of content shared on Facebook every month
40% projected growth in global data generated per year vs. 5% growth in global IT spending
235 terabytes data collected by the US Library of Congress in April 2011
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What is "Big" ?

Measuring Units

- 1 byte $\sim$ 1 character
- 10 bytes $\sim$ 1 word
- 100 bytes $\sim$ 1 sentence
- 1 kilobyte $=$ 1,000 bytes $\sim$ 1 page

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  $\sim$ a big shelf full of books
- 1 terabyte = 1,000 gigabytes
  $\sim$ all books in the Library of Congress
- 1 petabyte = 1,000 terabytes
  $\sim$ 20 million 4-door filing cabinets full of text

1 byte $\sim$ 1 grain of sand

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1 terabyte $\sim$ number of grains to fill a swimming pool

Rolling the Dice on Big Data

Data

Rolling the Dice on Big Data

Rolling the Dice on Big Data

Not quite

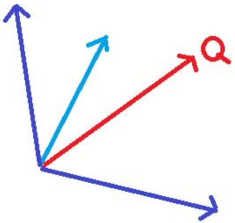
The Data in this Talk

Given:

Database: Collection of "documents" (data points)
Query: Single "document" (data point)

Want:

Documents closest to query



A tiny example to illustrate a "big data" problem

A "Tiny Data" Example

Database: Emails from known authors

Email 1: shipment of gold damaged in a fire
Email 2: delivery of silver arrived in a silver truck
Email 3: shipment of gold arrived in a truck

Query: Email from unknown author

gold silver truck

Which emails match the query best?
These emails may give clues about the author of query

Simplest approach for matching: **Word frequency**

Tabulating Emails and Query

Database (term document matrix) + Query

Terms	Email 1	Email 2	Email 3	Query
a	1	1	1	0
arrived	0	1	1	0
damaged	1	0	0	0
delivery	0	1	0	0
fire	1	0	0	0
gold	1	0	1	1
in	1	1	1	0
of	1	1	1	0
silver	0	2	0	1
shipment	1	0	1	0
truck	0	1	1	1

Basic Approach for Finding Matching Emails

1. Common words
   For each Email: Count number of words **common** to Email and Query

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2. Length
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3. Matching score for each Email:

$$\text{Matching score} = \frac{\text{Number of common words}}{(\text{Length of Email}) * (\text{Length of Query})}$$

Emails with highest matching scores:
May give clues about authors of Query

“Count” Common Words in Query and Email 1

Terms	E_1	Q	Multiply
a	1	0	0
arrived	0	0	0
damaged	1	0	0
delivery	0	0	0
fire	1	0	0
gold	1	1	1
in	1	0	0
of	1	0	0
silver	0	1	0
shipment	1	0	0
truck	0	1	0
Sum			1

common words in Email 1 and Query: $E_1 * Q = 1$

“Count” Common Words in Query and Email 2

Terms	E_2	Q	Multiply
a	1	0	0
arrived	1	0	0
damaged	0	0	0
delivery	1	0	0
fire	0	0	0
gold	0	1	0
in	1	0	0
of	1	0	0
silver	2	1	2
shipment	0	0	0
truck	1	1	1
Sum			3

common words in Email 2 and Query: $E_2 * Q = 3$

"Count" Common Words in Query and Email 3

Terms	E_3	Q	Multiply
a	1	0	0
arrived	1	0	0
damaged	0	0	0
delivery	0	0	0
fire	0	0	0
gold	1	1	1
in	1	0	0
of	1	0	0
silver	0	1	0
shipment	1	0	0
truck	1	1	1
Sum			2

common words in Email 3 and Query: $E_3 * Q = 2$

Basic Approach for Finding Matching Emails

1. Number of words **common** to Emails and Query

$$
\begin{aligned}
E_1 * Q &= 1 \\
E_2 * Q &= 3 \\
E_3 * Q &= 2
\end{aligned}
$$

2. Length
   Count **number of words** in each email, and in query

Length of Query

Terms	Q	Square
a	0	0
arrived	0	0
damaged	0	0
delivery	0	0
fire	0	0
gold	1	1
in	0	0
of	0	0
silver	1	1
shipment	0	0
truck	1	1
$\sqrt{\textbf{Sum}}$		$\sqrt{3}$

Length of Query: $\|Q\| = \sqrt{3} \approx 1.7$

Length of Email 2

Terms	E_2	Square
a	1	1
arrived	1	1
damaged	0	0
delivery	1	1
fire	0	0
gold	0	0
in	1	1
of	1	1
silver	2	4
shipment	0	0
truck	1	1
$\sqrt{\textbf{Sum}}$		$\sqrt{10}$

Length of Email 2: $\|E_2\| = \sqrt{10} \approx 3.2$

Basic Approach for Finding Matching Emails

1. Number of words **common** to Emails and Query

$$\begin{aligned} E_1 * Q &= 1 \\ E_2 * Q &= 3 \\ E_3 * Q &= 2 \end{aligned}$$

2. **Length** of Emails and Query

$$\begin{aligned} \|Q\| &= \sqrt{3} \approx 1.7 \\ \|E_1\| &= \sqrt{7} \approx 2.6 \\ \|E_2\| &= \sqrt{10} \approx 3.2 \\ \|E_3\| &= \sqrt{7} \approx 2.6 \end{aligned}$$

Matching Score for each Email

$$\text{Matching score} = \frac{\text{Number of common words}}{(\text{Length of email}) * (\text{Length of query})}$$

Email 1

$$\frac{E_1 * Q}{\|E_1\| \, \|Q\|} = \frac{1}{\sqrt{7}\,\sqrt{3}} \approx .22$$

Email 2

$$\frac{E_2 * Q}{\|E_2\| \, \|Q\|} = \frac{3}{\sqrt{10}\,\sqrt{3}} \approx .55$$

Email 3

$$\frac{E_3 * Q}{\|E_3\| \, \|Q\|} = \frac{2}{\sqrt{7}\,\sqrt{3}} \approx .44$$

Email 2 is the best match for the query

Conclusion for “Tiny Data” Example

Database: Emails from known authors

Email 1: shipment of gold damaged in a fire
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Query: Email from unknown author

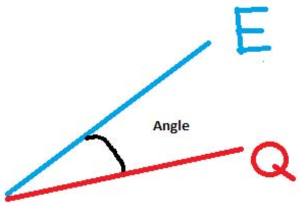
gold silver truck

Best matching email:

Email 2: delivery of silver arrived in a silver truck

The Reason for the Weird Way of Counting

Vector Space Model



Emails, Query = vectors

Matching score = cosine of **angle** between Email and Query

$$\frac{E * Q}{\|E\|\,\|Q\|} = \cos\angle(E, Q)$$

What this means “in practice”

Average number of emails per day: 294 billion
Number words in English language: at least 250,000

Matching one query with a **single** email:

250,000 operations (one for every possible word)

Matching one query with **all** emails:

250,000 * 294 billion $= 73.5 \cdot 10^{15}$ operations

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- Fast PC (Intel Core i7 980 XE)

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- Fast PC (Intel Core i7 980 XE)
  109 Gflops $= 109 * 10^9$ floating point operations per second
  Matching one query with all emails: about 8 days
- US supercomputer (Cray XT5, Opteron quad core 2.3GHz)
  Peak 1,381,400 Gflops
  Matching one query with all emails: about 1 minute

Can the Matching be Performed Faster?

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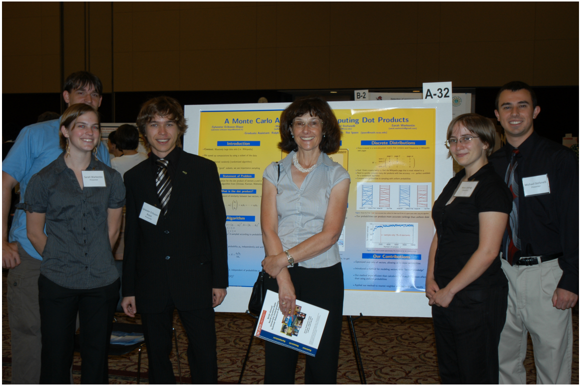
Yes!



Ralph Abbey, Sarah Warkentin, Sylvester Eriksson-Bique, Mary Solbrig, Michael Stefanelli

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on which words to use for the matching

Randomized Query Matching Algorithm

Idea

Do not use every word in query and emails

Monte Carlo Sampling: Use only selected words

{Downsize to smaller database with fewer words}

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Justification

- Don't need exact matching scores
  Identify only emails with highest matching scores
- Database available for offline computation
  Derive "statistics" based on word frequencies
- Perform query matching online
  Use "statistics" to select words used for matching

Suggestions for Downsizing the Database

- Statistics

  n: number of words in database
  Q_j: frequency of word j in query
  W_j: frequency of word j in database

- Suggestion for selecting word j

  Probability of sampling word j

$$p_j = \frac{W_j\, Q_j}{W_1\, Q_1 + \cdots + W_n\, Q_n}$$

Frequently occurring words more likely to be sampled

Rolling the Dice = Downsizing the Database

User input

s: number of samples

$\{$*number of words in downsized database*$\}$

Monte Carlo Sampling $\{$Roll the dice s times$\}$

For $t = 1, \ldots, s$

Sample *index* j_t *from* $\{1, \ldots, n\}$ **with probability** p_{j_t} *independently and with replacement*

Downsized database contains only s words:

word j_1, word j_2, ..., word j_s

Matching with Downsized Database

Downsized database: word j_1, word j_2, ..., word j_s

Word frequency in Query: $\hat{Q} = \begin{pmatrix} Q_{j_1} & Q_{j_2} & \dots & Q_{j_s} \end{pmatrix}$

For each Email E:

Matching with Downsized Database

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For each Email E:

- Word frequency $\hat{E} = \begin{pmatrix} F_{j_1} & F_{j_2} & \dots & F_{j_s} \end{pmatrix}$

Matching with Downsized Database

Downsized database: word j_1, word j_2, ..., word j_s
Word frequency in Query: $\hat{Q} = \begin{pmatrix} Q_{j_1} & Q_{j_2} & \ldots & Q_{j_s} \end{pmatrix}$

For each Email E:

- Word frequency $\hat{E} = \begin{pmatrix} F_{j_1} & F_{j_2} & \ldots & F_{j_s} \end{pmatrix}$
- Approximate number of words **common** to Email and Query

$$C = \frac{1}{s} \left(\frac{F_{j_1}\, Q_{j_1}}{p_{j_1}} + \frac{F_{j_2}\, Q_{j_2}}{p_{j_2}} + \cdots + \frac{F_{j_s}\, Q_{j_s}}{p_{j_s}} \right)$$

$\{s,\ p_{j_1},\ p_{j_2},\ \ldots,\ p_{j_s}$ compensate for fewer words$\}$

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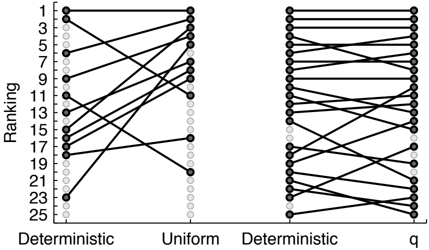
- Approximate **matching score** of Email: $\frac{C}{\|\hat{E}\|\ \|\hat{Q}\|}$

Reuters-215787 Collection: Transcribed Subset

201 documents and 5601 words
Number of sampled words $s = 56 \approx 1$ percent

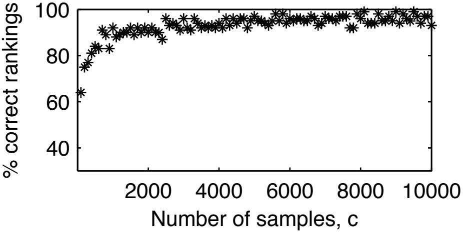


Bucket of computed 25 best matches contains
Correct 10 best matches in 99% of all cases

Wikipedia Dataset

200 documents and 198,853 words

Average percent of correct 10 best matches
as function of sample size



Sampling 1% of the words gives correct 9 best matches.
More sampling does not help a lot.

Summary

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Matching queries against document database

Rolling the dice

Randomized downsizing of database vocabulary
Frequently occurring words more likely to be kept

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Why use a randomized algorithm?

Advantages of randomized algorithm

- Easy to analyze
- Fast, and simple to implement
- As good in practice as deterministic algorithm
  (for this type of application)

The Bigger Picture

Many different methods for fast query matching

Algorithm in this talk:

Randomized matrix vector multiplication

Other randomized matrix algorithms:

Matrix multiplication
Subset selection
Least squares problems (regression)
Low rank approximation (PCA)

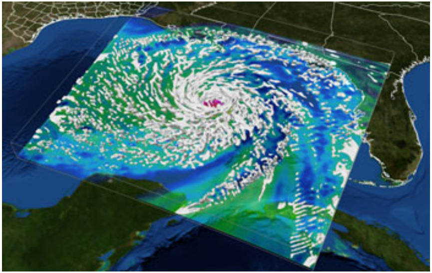
Applications for randomized algorithms:
Social network analysis, population genetics, circuit testing, ...

National Science Foundation, 29 March 2012

Press Release 12-060

NSF Leads Federal Efforts In Big Data

At White House event, NSF Director announces new Big Data solicitation, $10 million Expeditions in Computing award, and awards in cyberinfrastructure, geosciences, training



Hurricane Ike visualization created by Texas Advanced Computing Center (TACC) supercomputer Ranger.
Credit and Larger Version